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# NASA Technical Memorandum

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## ACTIVE CONTROL OF LARGE SPACE STRUCTURES: AN INTRODUCTION AND OVERVIEW

By George B. Doane III, Danny K. Tollison and  
Henry B. Waites

Systems Dynamics Laboratory

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## TECHNICAL MEMORANDUM

### ACTIVE CONTROL OF LARGE SPACE STRUCTURES: AN INTRODUCTION AND OVERVIEW

#### INTRODUCTION

Although large distributed systems are familiar to all, for example, highway bridges and roofs on houses, the need to consider this class of structure as a principle (or perhaps only) ingredient of a "plant" in a control system is just now emerging on a grand scale. This impetus comes from large aerospace structures intended for orbital use. The cost of orbiting anything is relatively large and, coupled with the fact of virtually free fall, i.e., near zero "g" conditions, produces an economic set of circumstances conducive to large-in-dimension yet extremely lightweight and, hence, limber structures. When this type of structure is limber enough so that it may not sensibly be characterized as a rigid body and, therefore, has to be modeled as an extended flexible body whose multitudinous vibration frequencies occur closely or densely packed and at least a good number of which fall within the range of control frequencies or required controller bandwidth, then the plant is classified as a Large Space Structure (LSS) for control purposes. Note that this latter definition tends to remove any absolute correlation with physical extent (although physical size was a powerful initial investigative motivator) and has moved the definition into the character of the plant dynamics and to the interaction of the plant dynamics with those of the controller.

#### DYNAMIC MODELING OF STRUCTURES

The authors' experience indicates that almost all the practitioners of the control design art spring from backgrounds other than those of the structural dynamicist. This being the case, there is the usual difficulty in communication between the two groups of practitioners. This difficulty appears to stem not only from differences in jargon but also from the type of mathematical modeling employed. We assume here that the general reader's background is not in structural dynamics. Thus, we will in the paragraphs below sketch some typical structural dynamical approaches to modeling.

Whereas control designers tend to start their design process with equations of the form,  $\dot{x} = Ax + Bu$  or  $Y(s)/X(s) = G(s)$ , depending upon whether a time or frequency domain approach is to be used, these equations, in the case of a LSS, typically represent the end of much travail on the part of the structural dynamicist. Indeed, left entirely to their own devices, they may never put the dynamical equations in quite this form although there is seldom a real problem in so doing.

The dynamicists' problems stem from consideration of extended material bodies characterized by non-lumped mass and stiffness properties and by generally little known energy dissipation characteristics. Indeed, the damping or energy dissipation of most structures is so ill defined that ad hoc damping additions to the equations of motion are often used after the equations are derived neglecting damping.

Two analytical approaches are in general use to produce the desired equations of motion, i.e., those equations casually relating actuators and sensors mounted on

the structure in question. For those who have not considered this arena we warn that questions of observability and controllability are crucial in structural control work. The two approaches in use are: (1) to model the extended body with partial differential equations (pde) and, (2) to model the extended body as a large collection of lumped masses and springs, i.e., finite element approaches as typified by the computer software package, NASTRAN. Whichever viewpoint is involved (and the choice is often arbitrary) the equations of motion are most often generated by use of energy methods, typically by use of Hamilton's Principle or La Grange's equations as contrasted to direct application of Newton's dynamical laws.

An example of the pde approach is the long slender beam, an oft-used extended structural element. Its homogeneous equation of motion may be shown to be

$$\frac{\partial^2}{\partial t^2} m(x) y(x,t) = \frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 u(x,t)}{\partial x^2} \right]$$

where  $m(x)$  describes the mass distribution along the beam,  $E$  and  $I$  are material and geometric properties, and  $y(x,t)$  is the transverse deflection of the beam as a function of displacement along the beam and as a function of time. Noting the absence of damping, one is led to try the Method of Separation of Variables to break the solution down into separate temporal and spatial descriptions. Letting  $y(x,t) = Y(x)f(t)$  and plugging away at the indicated operations yields

$$\frac{d^2}{dx^2} \left[ EI(x) \frac{d^2 Y(x)}{dx^2} \right] - \omega^2 m(x) Y(x) = 0$$

and

$$\frac{d^2}{dt^2} f(t) + \omega^2 f(t) = 0$$

Clearly the method yielded the correct results here as it does in various similar mathematical circumstances arising in electromagnetic theory, fluid and heat flow, etc. The solution of these equations gives rise to an infinite number of possible frequencies, i.e.,  $\omega$ 's (solved numerically from a transcendental equation and well tabulated [1]). Simultaneously for each  $\omega$  there corresponds a  $Y(x)$  or transverse deflection curve envelope along the beam. This procedure is a form of the linear systems. In this case  $Y(x)$  is the eigenvector or mode shape for a given frequency or eigenvalue,  $\omega$ . The total deflection is the sum of all the  $Y(x)$ . Having achieved the homogeneous solution, several approaches are available for determining forced responses. Space prohibits further exposition, but the reader is directed to the methods associated with Green's Function [2] for one approach. However, the reader is cautioned to return to the formulation of the pde in order to include any external forces acting on the beam.

The finite or lumped element approach may appeal conceptually to some in that it is analogous to lumped electric circuit theory. Just as distributed lead capacitance and inductance are often neglected and all such elements assumed lumped at one

physical spot in a circuit, so the beam, in this case, is modeled as a series of masses interconnected by flexures (springs). Depending upon the fidelity of the model, the masses may be modeled to rotate as well as translate. Conceptually, of course, one could apply D'Alembert's Principle to each mass in turn to generate the equations of motion. In actuality, La Grange's equation is most often used to produce a vector matrix total differential equation of the form

$$[m] \ddot{x} + [k] x = Q$$

where  $[m]$  is called the mass matrix,  $[k]$  the stiffness matrix and  $Q$  represents some generalized force input to the system. The transformation of this equation into the state variable form, i.e.,  $\dot{x} = Ax + Bu$ , is straightforward.

Solution of the homogeneous equation  $\dot{x} = Ax$  is, of course, another "eigenvalue" problem from which one discovers the eigenvalues (roots of the characteristic equation) or vibrational frequencies and the eigenvectors or ratios of the displacements of the masses relative to one another for any given eigenvalue. The absolute values of the displacements depend upon the fairly arbitrary initial conditions applied to the beam.

From whichever approach the equations are obtained, the equation of dimensionality looms large. Inherent in the pde formulation are an infinite number of degrees of freedom. In the lumped approach the number of states (in the control sense) is the sum of the number of generalized masses and springs. In a "realistic" model either number is often formidable. Indeed, this is a research issue at this time. One strives to use just enough but no more degrees of freedom in the design process than are required. Then one uses a proof model for validation incorporating a greater number of states than were used for controller design. (More on this later.) Of course the question arises, "How many degrees of freedom should be used in the proof model vis-a-vis the design model?" One approach for design is to use a model incorporating all structural frequencies anywhere near to and/or encompassing the controller bandwidth. Another approach is to retain all modes substantively causally relating the control system actuator(s) and sensor(s). A little reflection shows that this approach specifically neglects uncontrollable and/or unobservable modes regardless of frequency or amplitude. And why not? The control system can do nothing with them.

Another elegant but not widely used approach is due to Skelton [3]. In his Modal Cost Analysis (MCA) approach a scalar cost

$$J = \lim_{t \rightarrow \infty} E[y^T Q y + u^T R u]$$

is established (where  $E$  denotes the expected value,  $y^T Q y$  the "output cost" and  $u^T R u$  the "input cost") and the methodology of modern control techniques used to minimize  $J$  in such a way as to identify the major contributors to  $J$  regardless of their frequency or mode shape. As is generally the case, the choice of  $Q$  and  $R$  is arcane and especially so for this problem. As indicated earlier, this remains a research area.



## LSS CONTROL SYSTEM DESIGN

Discussion of the LSS control system design problem begins with the various models used for design and evaluation of performance in the controls area. Certain important LSS control system design objectives are put forth and finally a brief discussion is given concerning control law design techniques.

### LSS Models

The control system designer almost always begins the design process with a model of the plant to be controlled. Often this model is a simplified version of a more complicated one which has been reduced to make the design problem more tenable. Examples of this are the use of linearized models for design and the practice of neglecting the dynamics of certain system components which have high bandwidth compared to the system to be controlled. The simplification of models is carried to new lengths in LSS control system design.

As stated above, the exact description of a structure (or any distributed parameter system) in terms of linear ordinary differential equations requires infinite order of the model. This is obviously unreasonable for analysis and design purposes, so a so-called physical model [4] is produced. The physical model is the largest finite order model available. In the case of finite element modeling, the physical model is the untrimmed output of the modeling process. The size of the model depends upon the number of elements modeled in the structure. In the case of the beam described above in which an essentially infinite number of modes may be determined, engineering judgement must be used to determine how many are necessary to constitute the physical model. This is similar to the type of judgement which must be made to determine how many elements must be used in the finite element modeling process.

The physical model provides a starting point from which reasonable models may be chosen for control system design and evaluation. The evaluation or proof model is a subset of the physical model which maintains adequate fidelity (enough modes) for the testing of control concepts in simulation. This implies that it must also be small enough to be handled in a computer simulation. Of course, "adequate fidelity," depends upon the particular system requirements and, "small enough to handle," depends on the computer and the amount of CPU time available.

From the evaluation model comes the design model. The design model must include all modes which are expected to interact heavily with the control system. The maximum allowable size of the model is affected by the particular design technique(s) to be used and the "tools" available to carry out the design process.

The dynamical model, whether physical, proof, or design, is often presented to the controls engineer in the form

$$\ddot{\eta} + \Omega^2 \eta = LF$$

where  $\eta$  is a vector of generalized modal coordinates,  $\Omega^2$  is a diagonal matrix of the squares of the modal frequencies, and  $L$  is a matrix of eigenvalues which relate the



inputs,  $F$ , to the modes. Transformation of this system into the state variable form results in a block diagonal  $A$  matrix composed of second order blocks. Another useful form of this model is that of a set of underdamped second order pole pairs having common inputs and summed outputs. This form is particularly useful for frequency domain design techniques. Typically, some arbitrary damping (e.g.,  $\zeta = 0.005$ ) is added to the modal system when it is transformed for control use.

### Design Objectives

Once the designer has the appropriate model in hand, he must consider what is to be accomplished by the control system. One area which concerns the LSS control system designer is that of vibration suppression. As has already been pointed out, LSS's exhibit very low damping, i.e., a considerable amount of time may be required for a particular motion of the vehicle to damp out once it is excited. In the case of a precision pointing structure (such as a telescope), this may be a great design concern. In this case, the designer may choose to determine which motion of the structure is causing problems and place rate sensors (accelerometers or rate gyros) and force (or torque) actuators (proof mass forcers or control moment gyros) so as to damp this motion.

Shape control is another area of interest in the study of LSS's. Large orbiting antennae provide an application of shape control. Forces as small as those imposed by the gravity gradient field and thermal stress are adequate to deform such structures beyond the limits of their performance requirements. Shape control systems employ position sensors (often optical devices) and position actuators such as cord pullers or piezoelectric pushers and are usually of relatively low bandwidth.

A third area of great concern to the LSS control system designer is disturbance isolation. This area involves isolating certain sensitive parts of a structure (the optics of a telescope, for instance) from disturbances transmitted into them from other more "noisy" parts of the vehicle. This is among the most challenging of LSS control problems.

### Design Techniques

Many techniques are proposed for control law design for LSS's but few are proven. Many of these are based on the familiar state variable approach to the problem. This is inviting because LSS control systems are almost always multi-input/multi-output (MIMO). Among these schemes are pole placement techniques, optimal output feedback schemes and others. Various frequency domain techniques are also proposed which surmount some of the problems involved with applying classical techniques to MIMO systems [5].

### PROBLEM AREAS

The problem areas in LSS control system design are many and challenging. Foremost among these is model uncertainty. Assuming that a very good physical model is available to the designer, it is likely that the design model will be in significant error because of the model reduction process. This is, of course, a function of how well the design model was chosen. At any rate, the use of a truncated model demands robustness from the control system.

In addition to the error incurred because of model reduction, there may be significant error in the physical model from which the design model was generated. Error in modal frequency values of 20 to 100 percent is not unusual, but the error is typically less for the lower modal frequencies than for the higher ones which usually presents an advantage for the control designer. This uncertainty of high frequency modes leads the designer to gain stabilize them in most cases where the lower frequency modes may be phase stabilized.

The error incurred due to the use of truncated models reveals itself as control and observation spillover. Control spillover is structural reaction due to the excitement of modes by the control system which were not included in the design model while observation spillover is sensor output due to excitement of such modes. Spillover is a major motivator in the use of the proof model for control system verification.

A final problem in the study of LSS's which lies along quite different lines to those already discussed, but concerns all facets of the modeling and design areas, is that of verification of the theory on actual LSS hardware. The very nature of LSS's makes them very difficult to study in the one "g" environment of earth. Indeed, many LSS designs would fail to support themselves on earth, were they actually constructed and deployed here without extensive extra support mechanisms. Orbital tests, on the other hand, are very expensive for any full scale LSS and the risks for the payoff seem quite high. Eventually, a careful blend of on orbit tests, preceded and supported by carefully contrived ground based tests, most likely will produce the most efficient results in the proof of LSS control and modeling concepts. To this end, work is currently being carried out to develop a full scale LSS test facility at the NASA Marshall Space Flight Center [6].

## CONCLUSIONS

A brief overview of the Large Space Structure (LSS) modeling and control problem has been presented from the control engineering point of view. The LSS has been defined and techniques for developing dynamical models of such systems discussed. It is clear that reduced order models must be used in LSS control system design while the matter of how to choose such models remains a research issue at this time. Typical LSS control system objectives are: vibration suppression, vibration isolation, and shape control. Many promising LSS control law design techniques have been proposed by the research community, including candidates from both the time and frequency domains. A discussion of such LSS pitfalls as model uncertainty, model truncation, and spillover has been presented and finally a case has been made for the development of ground based testing facilities for LSS's.

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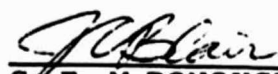
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## APPROVAL

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By George B. Doane, III, Danny K. Tollison,  
and Henry B. Waites

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.



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G. F. McDONOUGH

Director, Systems Dynamics Laboratory